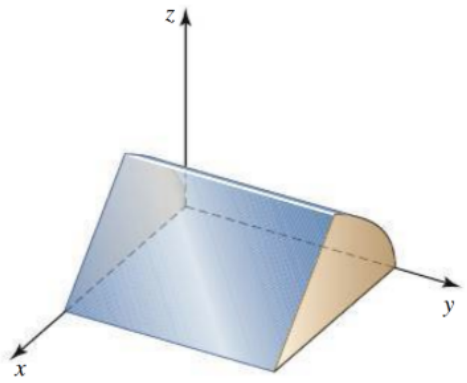
Triple Integrals

Integrals of functions of 3 or more variables in no different than of 2. The differential is usually denoted dV.

Example 16.4.1. Compute the integral $\int_0^{\pi/2}\int_0^1\int_0^{\pi/2} \sin \pi x \cos y \sin 2z dy\, dx\, dz$.

Example 16.4.2. Find the volume of the wedge in the first octant bounded by the cylinder $x = z^2$ and the planes $z = 2 - x, y = 2, y = 0$ and $z = 0$.



Example 16.4.3. Evaluate the integral $\int_1^4 \int_z^{4z} \int_0^{\pi^2} \frac{\sin\sqrt{yz}}{x^{3/2}} dy\, dx\, dz$.

Transformations Between Cylindrical and Rectangular Coordinates

Rect. → Cyl.	**Cyl. → Rect.**
$r^2 = x^2 + y^2$	$x = r\cos\theta$
$\tan\theta = y/x$	$y = r\sin\theta$
$z = z$	$z = z$

$$dv \to dz\, r\, dr\, d\theta$$

Transformations Between Spherical and Rectangular Coordinates

Rect. → Sph.	**Sph. → Rect.**
$\rho^2 = x^2 + y^2 + z^2$	$x = \rho\sin\varphi\cos\theta$
trig to find φ and θ	$y = \rho\sin\varphi\sin\theta$
	$z = \rho\cos\varphi$

$$dv \to \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$$

Example 16.4.4. Evaluate $\iiint_D (x^2 + y^2 + z^2)^{-3/2} dV$ where D is the region in the first octant between two spheres of radius 1 and 2 centered at the origin.

Example 16.4.5. Evaluate the integral using cylindrical coordinates:

$$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_0^{9-3\sqrt{x^2+y^2}} dz\, dx\, dy$$

Example 16.4.6. Find the volume of the solid lying between the planes $z = 1$ and $z = 2$ that is bounded by the cone $z = \sqrt{x^2 + y^2}$. Use spherical coordinates.